

transformation methods. The main reason for this inefficiency is the requirement of exact minimization of the transformation function at each iteration. The methods are quite robust as is the RQP method of Pshenichny¹ (a primal method). The RQP method converges to a local minimum point that is closest to the starting point. This motivates the development of hybrid methods where a large step algorithm may be used in the beginning to skip over certain local minima. Hybrid methods also are motivated by the desire to combine efficiency with reliability.

It is concluded that results from the study should form a basis for future developments in computational methods for optimum structural design.

Acknowledgment

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Reference

¹Belegundu, A. D. and Arora, J. S., *A Study of Mathematical Programming Methods for Structural Optimization*, Materials Engineering Division, The University of Iowa, Iowa City, Ia., Tech. Rept. CAD-SS-82. 5, (available from National Technical Information Service), Aug. 1982.

Supersonic Flutter of Short Panels on an Elastic Foundation

G. Venkateswara Rao* and K. Singa Rao†
Vikram Sarabhai Space Center, Trivandrum, India

Introduction

SUPERSONIC flutter of panels is the self-excited oscillation of the external skin of a flight vehicle exposed to an airflow along its surface. Reference 1 gives an excellent review of this topic. In Refs. 2-4 the finite element method has been successfully used to investigate the flutter of panels. The effects of shear deformation and rotatory inertia on the flutter of panels has been studied in Ref. 5.

In this Note the effect of an elastic foundation on the supersonic flutter of short panels is studied using the finite element method. The formulation used takes care of the effects of shear deformation and rotatory inertia. Flutter parameters are obtained for simply supported, simply supported and clamped, and clamped panels and the results are presented in tabular form.

Finite Element Formulation

The panel (Fig. 1) is represented by a flat plate of unit width in bending. The supersonic airstream flows over the upper surface of the plate in the positive X -direction and the bottom surface is supported on an elastic foundation.

The matrix equation governing the motion of the panel is

$$[K]\{q\} + \lambda[A]\{q\} + \gamma[F]\{q\} - \Omega[M]\{q\} = 0 \quad (1)$$

where $[K]$ is the stiffness matrix including shear deformation, $[A]$ the aerodynamic matrix, $[F]$ the foundation matrix, $[M]$ the mass matrix including rotatory inertia, λ the

dynamic pressure parameter, γ the foundation parameter, Ω the nondimensional eigenvalue parameter, and $\{q\}$ the eigenvector.

The nondimensional quantities λ , γ , and Ω are defined as

$$\lambda = 2qL^3/D(M_\infty^2 - 1)^{1/2} \quad (2)$$

where q is the dynamic pressure, L the length of the panel, D the panel flexural rigidity, and M_∞ the Mach number;

$$\gamma = kL^4/D \quad (3)$$

where k is the foundation modulus per unit area; and

$$\Omega = \frac{M_\infty^2 - 2}{M_\infty^2 - 1} \frac{L}{U} \nu - \frac{mL^4}{D} \nu^2 \quad (4)$$

where U is the flow velocity, ν the flexural frequency, and m the mass per unit area.

The matrices $[K]$, $[M]$, and $[A]$ are assembled from the element matrices $[k]$ and $[m]$ taken from Ref. 6 and $[a]$ derived following the method proposed by Olson.^{3,4}

Table 1 Values of λ_{cr} and Ω_{cr} for a simply supported panel

L/t	γ	λ_{cr}	Ω_{cr}
50.0	0.0	342.0	1047
	1.0	342.0	1048
	10.0	342.0	1057
	100.0	342.0	1147
	1000.0	342.0	2047
25.0	0.0	338.5	1036
	1.0	338.5	1037
	10.0	338.5	1046
	100.0	338.4	1135
	1000.0	337.4	2032
10.0	0.0	315.0	962.9
	1.0	315.0	963.9
	10.0	315.0	972.9
	100.0	314.3	1060
	1000.0	308.1	1934
5.0	0.0	248.4	755.1
	1.0	248.4	756.1
	10.0	248.0	763.9
	100.0	245.6	843.8
	1000.0	220.5	1642

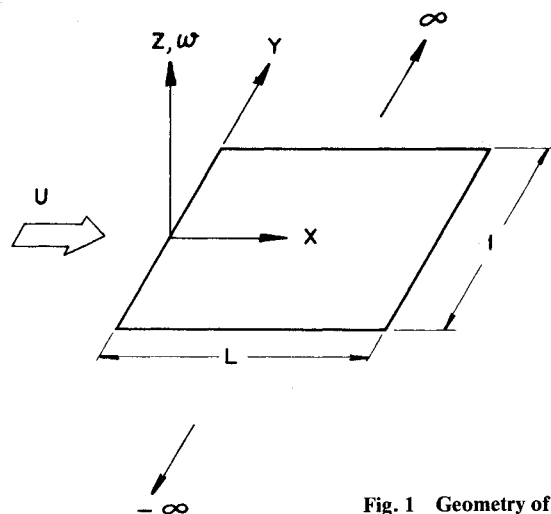


Fig. 1 Geometry of the panel.

Table 2 Values of λ_{cr} and Ω_{cr} for a simply supported and clamped panel

L/t	γ	λ_{cr}	Ω_{cr}
50.0	0.0	474.6	1726
	1.0	474.6	1727
	10.0	474.6	1736
	100.0	474.6	1826
	1000.0	474.6	2726
25.0	0.0	465.8	1694
	1.0	465.8	1695
	10.0	465.8	1704
	100.0	465.6	1793
	1000.0	465.6	2689
10.0	0.0	410.2	1491
	1.0	410.2	1492
	10.0	410.2	1501
	100.0	409.5	1588
	1000.0	403.3	2460
5.0	0.0	279.6	1006
	1.0	279.6	1007
	10.0	279.2	1014
	100.0	276.8	1094
	1000.0	252.4	1888

Table 3 Values of λ_{cr} and Ω_{cr} for a clamped panel

L/t	γ	λ_{cr}	Ω_{cr}
50.0	0.0	625.6	2687
	1.0	625.6	2688
	10.0	625.6	2697
	100.0	625.6	2787
	1000.0	625.6	3687
25.0	0.0	606.9	2610
	1.0	606.9	2611
	10.0	606.9	2620
	100.0	606.6	2709
	1000.0	605.8	3604
10.0	0.0	499.1	2154
	1.0	499.1	2155
	10.0	499.1	2164
	100.0	498.4	2250
	1000.0	492.0	3120
5.0	0.0	290.7	1246
	1.0	290.7	1247
	10.0	290.5	1253
	100.0	288.0	1332
	1000.0	263.8	2122

The matrix $[F]$ is obtained by assembling the element matrices $[f]$ derived from the energy expression \bar{U} given by

$$\bar{U} = \frac{\gamma}{2} \int_0^l w^2 dx \quad (5)$$

where l is the element length.

It should be noted that the approximate aerodynamic theory used here is valid only for $M_\infty > 1.5$.

Numerical Results

Equation (1) is solved by using any standard algorithm to obtain the eigenvalues and eigenvectors. In the present study for a given γ and L/t (where t is the thickness of the panel), an eight-element idealization is used that gives accurate results for the flutter problems as shown in Ref. 7. The value of γ_{cr} is obtained where the two lowest nondimensional eigenvalue

parameters, Ω_1 and Ω_2 , coalesce to Ω_{cr} . This is repeated to obtain λ_{cr} and Ω_{cr} for different values of γ and L/t .

Using the procedure described above, values of λ_{cr} and Ω_{cr} at γ of 0, 1.0, 10.0, 100.0, and 1000.0 and L/t of 50.0, 25.0, 10.0, and 5.0 are obtained for simply supported, simply supported and clamped, and clamped panels. The results are presented in Tables 1-3.

Conclusions

Based on the present study of the supersonic flutter of panels resting on an elastic foundation, the following conclusions can be drawn:

1) For a given L/t and γ , the values of λ_{cr} and Ω_{cr} increase as one goes from simply supported to simply supported and clamped and to clamped panels.

2) For thin panels (i.e., for $L/t = 50$), λ_{cr} is the same for any value of γ and Ω_{cr} increases by the value of γ compared to the value of Ω_{cr} for $\gamma = 0$.

3) For a given γ , as L/t decreases, the values of λ_{cr} and Ω_{cr} decrease.

4) For thicker panels, for a given L/t the values of λ_{cr} decrease as γ increases. Thus, for thicker panels, the additional foundation stiffness has a destabilizing effect.

5) Even for thicker panels with L/t as low as 5, the value of λ_{cr} remains constant up to $\gamma = 1.0$ and the shift in Ω_{cr} is equal to the value of γ compared to the value Ω_{cr} for $\gamma = 0.0$.

6) The effect of shear deformation and rotatory inertia is significant for values of $L/t < 10.0$.

7) All the above conclusions hold for the three types of boundary conditions considered in the present study.

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Thermal Resistance of Circular Cylinder Cross Sections with Convective and Flux Prescribed Boundaries

G.E. Schneider*

University of Waterloo, Waterloo, Ontario, Canada

Introduction

IN the thermal analysis of fuel rod interaction with containment tubing,¹ in the analysis of double-walled heat exchangers which employ longitudinally grooved tubing as

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*Associate Professor of Mechanical Engineering. Member AIAA.